ZULFAQAR Journal of Defence Science, Engineering & Technology
e-ISSN: 2773-5281
Vol. 3, Issue 2 (2020)
DOI: https://doi.org/10.58247/jdset-2020-0302-07
Journal homepage: https://zulfaqarjdset.upnm.edu.my

# PATH FINDING OF STATIC INDOOR MOBILE ROBOT VIA AOR ITERATIVE METHOD USING HARMONIC POTENTIALS

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**ARTICLE INFO**

**ARTICLE HISTORY**
Received: 01-06-2020
Revised: 15-07-2020
Accepted: 30-09-2020
Published: 31-12-2020

**KEYWORDS**
Autonomous Navigation
Collision-Free
Finite Difference Scheme
Numerical Analysis
Optimal Path

**ABSTRACT**

Mobile robots often have to discover a path of collision-free towards a specific goal point in their environment. We are trying to resolve the mobile robot problem iteratively by means of numerical technique. It is built on a method of potential field that count on the use of Laplace's equation in the mobile robot's configuration space to constrain/which reduces the generation of a potential function over regions. This paper proposed an iterative approach in solving robot path finding problem known as Accelerated Over-Relaxation (AOR). The experiment shows that this suggested approach can establish a smooth path between the starting and goal points by engaging with a finite-difference technique. The simulation results also show that a more rapidly solution with smoother path than the previous work is achieved via this numerical approach.

## 1.0 INTRODUCTION

Path finding or navigation problem plays an essential role in the autonomous mobile robots. To build an autonomous mobile robot, it needs to be able to design a route from initial to the final configurations efficiently and reliably without colliding with any obstacles between them. Even in the fields for instance industrial robotics, computer animation, automated surveillance and drug design, the efficient algorithm also have important applications for solving these problems. It is therefore not surprising that over the past two decades the research in this area has gradually increased.

This paper aims at simulating a point-robot path finding in selected configuration space over numerical potential function constructed by means of the heat transfer theory. This model of heat transfer constructs a field that is free from local minima as well as helpful for directing the robot navigation. For this work, the heat transmission problem is displayed by means of the equation of Laplace's. The harmonic functions are also referred to as the Laplace's equation solution, which subsequently signify the temperature values in the configuration space to be used for the path generation simulation. Many methods have been used to accomplish harmonic functions; however, the most general way is by numerical techniques, owing to the convenience of fast computing machine and its neat and competence in problem solving. In this paper, some experiments were carried out to test the reliability of accelerated iterative method in producing mobile robot paths for different environment sizes.

## 2.0 RELATED STUDY

Through their pioneer work, Connolly and Gruppen [1] proved that the harmonic functions had a variety of useful characteristics through robotics. Meanwhile, Khatib [2] uses potential functions in robot path finding, where each obstacle generates repelling force whilst the targets employ an attractive force. In comparison, Koditschek [3] found that in certain types of domains, at least geometrically, there are

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potential functions which can assist the effector from almost any point in a certain region. The core shortcoming of these potential fields, though, was that they undergo the formation of local minima.

Connolly *et al*. [4] and Akishita *et al*. [5] have developed independently a global method which uses Laplace's equations solutions to induces smooth paths and compute the potential fields globally across the entire region. Both works show that harmonic functions provide a quick way to construct paths in a robot configuration space and prevent local minima from being created spontaneously. Previous works [6-10] showed that block methods execute much more rapidly than ordinary iterative methods of Jacobi and Gauss-Seidel. Some other approaches in solving the problem of path finding were also proposed. For instance, Willms and Simon [11] demonstrated an algorithm that employed the distance transform method. Whereas Jan *et al*. [12] carried out work on the application of the method of geometry maze routing algorithm. While Bhattacharya and Gavrilova [13] developed a method based on Voronoi Diagram. Furthermore, an approach focused on genetic algorithm was described in [14].

## 3.0 METHODOLOGY

Instead of using the real robot vehicle, we simulate the concept of robot vehicle movement using a point that moves in a known environment. The robot's path finding problem can be described as a heat transfer problem with a steady state. In the analogy of the heat flow, the target is viewed as a sink pulling heat in. The borders of configuration space and the obstacles are known as heat sources, by constant temperature values are set. Through the cycle of thermal conduction, the temperature dispersion extends, and the heat streamlines flows into the sink fillings the workspace. These areas can be described as communication mediums between the robot, goal and obstacles. The temperature dispersion is used as a director for mobile robot traveling from any starting to the target point by following the heat stream that flows through the configuration space from high temperature sources to the lowest temperature point. The temperature dispersion of the environment is determined by engaging harmonic function to model the setup of the environment.

Mathematically, a harmonic function on a domain $\Omega \subset R^n$ is a function that fulfils the Laplace's equation, in which $x_i$ is the *i*-th Cartesian coordinates and *n* is the dimension. In the case of constructing a robot path, the domain $\Omega$ contains of starting points, goal point, the outer boundary walls and all obstacles in the zone.

$$\nabla^2 \phi = \sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial x_i^2} \tag{1}$$

Laplace's equation can be solved professionally by means of numerical method. Jacobi and Gauss-Seidel are the standard methods used to tackle the problem, while Equation (1) has been solved in this paper using accelerated iterative technique for faster computation.

Within this model, in the configuration space a point is represented at the robot. The configuration space is constructed in a grid system. To comply with Equation (1), by using numerical technique, the function values connected with each node is then iteratively figured. The highest temperature is allocated to the starting point while the lowest is given to the target point. In the meantime, for the outer wall boundaries and obstacles, different initial temperature values are set. There is no requirement to give initial temperature values at the starting points. The Laplace's equation solutions were analysed with boundary conditions of Dirichlet, $\Phi \,|\, \partial\Omega = c$, where *c* is constant. The smooth path can be created by mapping the temperature dispersion via steepest descent technique once the potential values of the environment have been reached, where the algorithm follows the negative gradient from the starting point to the lowest temperature target point through successive points with lower temperature.

## 4.0 FORMULATION OF ACCELERATED OVER-RELAXATION ITERATIVE METHOD

In solving problem in Equation (1) based on the robotics literature, the standard GS [4] and SOR [15,16] were used. The computation for Laplace's equation (1) solution for this analysis is by using faster numerical solver i.e. Accelerated Over-Relaxation (AOR) iterative technique. Consider the equation of two-dimensional Laplace's in Equation (1) be defined as

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$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0 \tag{2}$$

The approximation of Equation (2) can be generalized through the five-point second-order standard finite difference formula as commonly defined in the next equation

$$U_{i-1,j}+U_{i+1,j}+U_{i,j-1}+U_{i,j-1}-4U_{i,j}=0 \tag{3}$$

To boost up the convergence speed, AOR iterative scheme is added to Equation (3) and the standard five-point formula of AOR can be written as

$$u_{i.j}^{(k+1)}=\frac{r}{4}\left(u_{i-1.j}^{(k+1)}-u_{i-1.j}^{(k)}-u_{i.j-1}^{(k+1)}-u_{i.j-1}^{(k)}\right)+\frac{\omega}{4}\left(u_{i-1.j}^{(k)}+u_{i+1.j}^{(k)}+u_{i.j-1}^{(k)}+u\right)+(1-\omega)u_{i.j}^{(k)} \tag{4}$$

where $r$ and $\omega$ are the parameters of optimum relaxation. The uncertain optimum values of $r$ and $\omega$ gave no restriction in getting the minimum number of iterations. Hadjidimos [17] stated that the value $r$ is usually selected to be near to the value $\omega$ of the corresponding SOR, where $1 \le \omega \le 2$.

## 5.0 EXPERIMENTS AND RESULTS

The design of the environment contains of four different sizes: 300x300, 600x600, 900x900 and 1200x1200. The target point was set with a fixed and lowest temperature values, although no specific temperature values were given for all three starting points. Various numbers of obstacles of various shapes were mounted in the environment. Initially, Dirichlet boundary condition was applied with high temperature values are fixed to the walls and the obstacles. All other points were set to zero temperature, apart from the target point set to the lowest temperature values.

The computation development was conducted on AMD A10-7400P Radeon R6, 10 Compute Cores 4C+6G operating at 2.50GHz speed with 8GB of RAM. The iteration process to calculate temperature values numerically at each point continues until the stopping criteria is encountered. If the temperature values have no more changed and the difference of computation values was greatly small, i.e. $1.0^{-10}$, the loop is terminated. This high precision was required to escape flat areas in the solutions, also known as saddle points, which can be causing the formation of path to flop.

Tables 1 and 2 show the number of iterations and execution time (in seconds), respectively, for all the numerical techniques compared in the experiment. The AOR iterative method was obviously very fast in comparison with the SOR method.

Table 1. Performance of the considered methods in terms of number of iterations

| Methods | | N x N | | | |
|---|---|---|---|---|---|
| | | 300 x 300 | 600 x 600 | 900 x 900 | 1200 x 1200 |
| Case 1 | SOR | 1728 | 8117 | 17831 | 31346 |
| | AOR | 1591 | 7529 | 16594 | 28984 |
| Case 2 | SOR | 2228 | 8776 | 19254 | 33558 |
| | AOR | 2006 | 7973 | 17538 | 30573 |
| Case 3 | SOR | 3624 | 14644 | 33004 | 57484 |
| | AOR | 3236 | 13165 | 29680 | 51738 |
| Case 4 | SOR | 2507 | 9868 | 21654 | 37762 |
| | AOR | 2288 | 9025 | 19840 | 34601 |

Table 2. Performance of the considered methods in terms of execution time (in seconds)

| Methods | | N x N | | | |
|---|---|---|---|---|---|
| | | 300 x 300 | 600 x 600 | 900 x 900 | 1200 x 1200 |
| Case 1 | SOR | 8.13 | 227.95 | 1134.25 | 3728.92 |
| | AOR | 8.61 | 230.17 | 1148.87 | 3692.74 |
| Case 2 | SOR | 10.69 | 251.72 | 1270.23 | 4077.22 |
| | AOR | 10.27 | 248.24 | 1226.66 | 3976.33 |
| Case 3 | SOR | 16.22 | 427.27 | 2190.45 | 7432.68 |

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| | | | | | |
|---|---|---|---|---|---|
| | AOR | 18.66 | 418.45 | 2073.25 | 7254.02 |
| Case 4 | SOR | 11.02 | 281.85 | 1441.47 | 4853.57 |
| | AOR | 12.52 | 281.78 | 1423.54 | 4743.21 |

The path has been created by executing steepest descent search from start to the target point once the temperature values have been achieved. From the current point, the paths creation process was formed very quickly, in which the algorithm simply chose the lowest temperature value from its neighbouring points. This cycle goes on till the target point is reached. Fig. 1 shows the routes successfully created in an obstacle setting based on the temperature dispersion profile gained through numerical computation. All starting points (square point) positively ended at the specific target point (round point), and avoided all the various obstacles locate in the place.

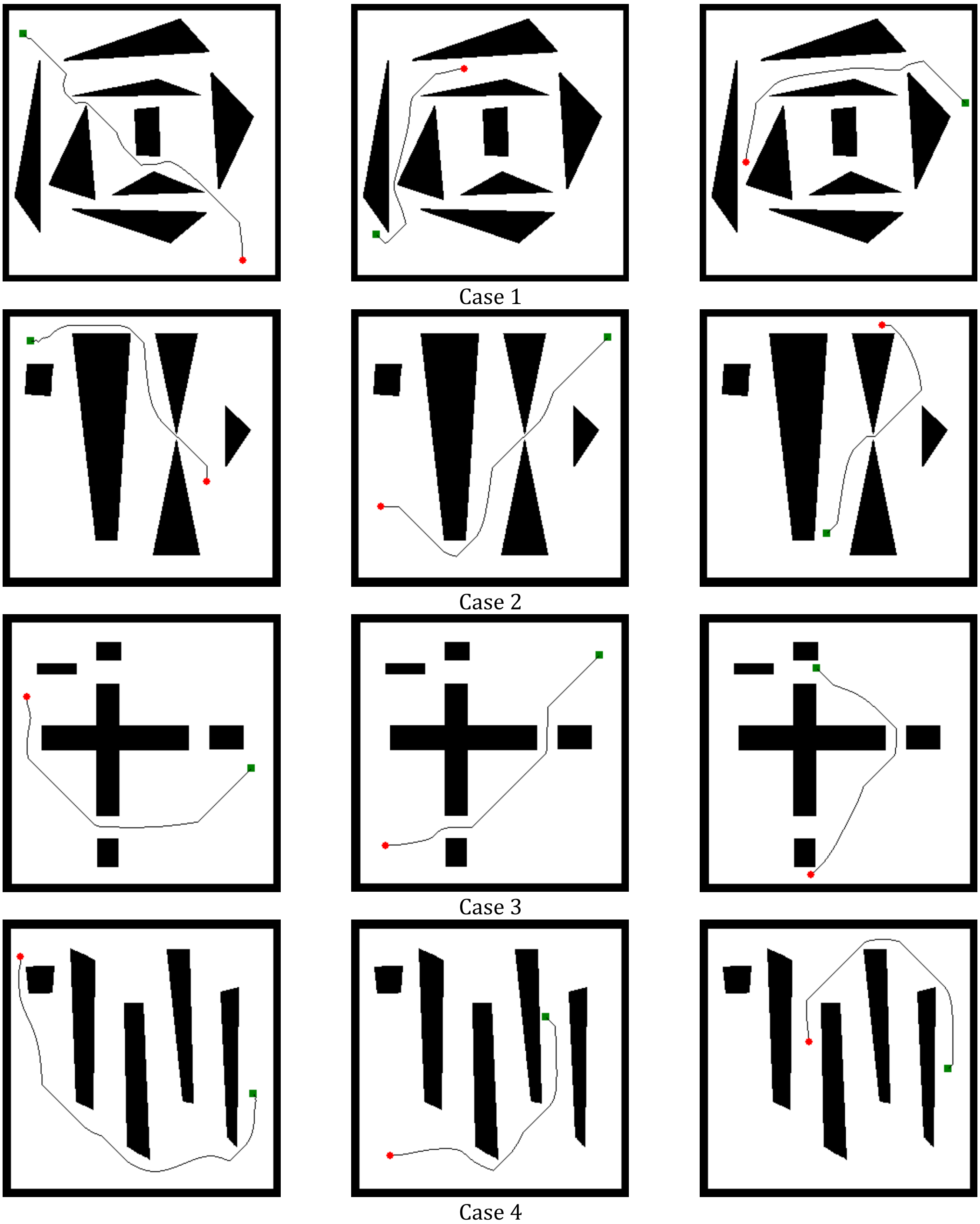


Figure 1. The generated paths from several different start (green dot/square dot) and goal (red dot/circle dot) positions for various environment

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## 6.0 CONCLUSION

The experiment of this study demonstrates that overcoming the problems of robot path finding through numerical techniques is still very appealing and attainable due to the newly developed and advanced techniques, and the availability of fast machines today. The iterative method of AOR evidenced to be very promptly comparing to the standard SOR method, as shown in the results table. The growth number of obstacles does not adversely affect the efficiency; the calculation simply gets faster as the occupied region by obstacles are ignored during computation. The results of this paper can essentially be categorized as family of full-sweep iterations. In addition to the full-sweep iteration concept, further analysis on half-sweep [15,18-21] and quarter-sweep [22-25] iterations can be also considered with a view to speeding up the convergence rate of the standard proposed iterative methods.

## 7.0 ACKNOWLEDGEMENT

The authors acknowledge National Defence University of Malaysia for the funding of this article. The researchers declare that there is no conflict of interest regarding the publication of this study.

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