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JOINT SEGMENTATION AND REGISTRATION VIA VARIATIONAL FORMULATION FOR 2D MONO-MODAL IMAGES

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ABSTRACT

Two of the main branches of image processing are segmentation and registration. Image segmentation aims to partition the images into foreground and background based on distinguishable characteristics, meanwhile, image registration involves finding an optimal geometric transformation from the given images. These two tasks are often treated separately, however, the joint models between segmentation and registration have their advantages compared to the separate tasks. In this paper, two variational models for joint image segmentation and registration are reviewed and compared using mono-modal images. Results showed that joint image segmentation and registration model based on linear curvature gives better results compared to the state-of-the-art models.

1.0 INTRODUCTION

Image segmentation and registration are two crucial steps in image processing. They have broad applications in medical imaging, computer vision, security systems, remote sensing, etc. Segmentation procedures divide the images into their constituent parts according to their features and properties. There exist many methods for image segmentation such as thresholding, edge-based and region-based segmentation. Edge based image segmentation methods can locate pixels near edges but are at a loss to locate exact edges. In addition, edge detection methods often produce discontinuous boundaries of the objects of interest. Meanwhile, the region-based segmentation methods consider more pixels than the edges to segment the images since the methods use a similarity measure. Furthermore, the methods produce closed boundaries of the objects of interest.

In image registration, two images are given, reference and template. The reference or so-called "the fixed image" is kept unchanged during the registration process. Meanwhile, the template or so-called "the moving image" is transformed using the found transformation to produce the transformed template image. The goal is to align the template image so that it appears like the reference image. Image registration models are widely used for inter- and intra-subject morphological comparisons, the creation of population atlases, the delivery of precision therapies and other areas in medical image processing.

Variational image processing models have been widely used in the last two decades, in image denoising, segmentation, surface reconstruction and registration [1]. The variational models, also known as the partial differential equations-based methods consist of several tools such as variational functional, energy optimization, regularization, numerical algorithms, etc. The focus of this paper is on region-based segmentation and registration models for mono-modal images using variational formulation. Mono-modal images refer to the images produced using the same imaging machines. Therefore, the same

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objects or features in the reference and template images have the same intensity values. They are directly comparable which allows the use of the mean squared difference (msd) as the performance measure. In contrast with the multi-modal images, where the template and reference images are produced from the different imaging machines. In this paper, eight pairs of mono-modal images from the Mouse Brain Gene Expression Data are used for the experiment. The data are provided by the Centre for Computational Biology, UCLA.

Normally, these segmentation and registration tasks are tackled independently. But the segmented features from the template image correspond to each other in the reference image. This fact provides a natural initial guess for transformation. In addition, image registration can pass on segmented boundaries from the template image to the reference image. Thus, the effectiveness of the two tasks can be improved significantly by coupling these two tasks. In addition, these two are dependent on each other. Any errors produced from segmentation will be prolonged to the registration and vice versa. Therefore, numerous approaches to combine these two techniques as a joint model have been developed to overcome their limitations. In 2011, Le Guyader and Vese presented a joint segmentation and registration (JSR) model in which the template and reference images are indirectly represented using level set functions for topology preservation [2]. They used a nonlinear elastic principle as the regularization term, allowing more significant deformation. Later, in 2014, Ozeré and Le Guyader enhanced the previous distance measure in [2] by introducing a new similarity measure based on weighted total variation (TV) to handle large classes of images [3]. Both methods are used to evaluate 2D mono-modal Mouse Brain Gene Expression Data.

Apart from that, Hodneland et al. initiated a novel JSR model approach for 4D dynamic contrast enhanced MRI (DCE-MRI) of the kidneys [4]. In this model, pre-segmenting processes use a time-normalized and continuous Mahalanobis distance as a training set. The training set is used to generate the training mask, which guides the actual segmentation problem referred to as supervised segmentation. Since the DCE-MRI kidney images contain reliable edge information between various tissue types hence, they applied Normalized Gradient Field (NGF) as a different approach to typical DCE-MRI registration that used mutual information (MI) or normalized mutual information (NMI) as the registration model. However, the weaknesses of this model are the requirement of a training set and shape information.

On the other hand, Mahapatra et al. proposed the usage of sparse data into joint segmentation and groupwise registration on cardiac DCE-MRI [5]. To determine the registration and segmentation cost, the authors disintegrated the intensity time signal of DCE-MRI data into sparse and low-rank components. The images were then segmented using K-mean clustering and registered using groupwise image gradients. A groupwise approach was implemented to impose constraints based on temporal flow for registration purposes. In 2016, Ibrahim et al. improved the JSR model from [2] by adding a weighted Heaviside sum of the squared difference (SSDH) into the model and replacing the previous regularization term with the linear curvature model introduced by Fischer and Modersitzki to produce smoother transformation compared to the nonlinear elastic model [6-7]. They showed that their proposed model can register images when the features inside the object of interest posed different kinds of deformations to the object itself. Unlike previous studies, Aganj and Fischl used the reference and template image segmentation errors as the registration distance measure [8]. However, they considered only rigid registration, which allows global rotation and translation deformation. Meanwhile, the deformations considered in [6] are non-rigid.

Aside from that, Swierczynski et al. proposed a novel JSR model using the level- set framework where the authors combined the classic Chan–Vese segmentation algorithm with a non-linear intensity-based registration [9-10]. The presented model was applied to 3D lung CT scans. They then compared the performance of their joint model with separate segmentation and registration tasks highlighted the advantages of joint work. Firstly, the JSR model will eliminate the problem of initializing a surface-based segmentation method. Secondly, the joint model will incorporate prior knowledge about the segmentation of the images into registration. Thirdly, the separate tasks of segmentation and registration are more prone to image noise and algorithm initialization. However, the results of their JSR model are still inferior to those achieved by the current state-of-the-art lung registration methods when applied to the Dir-Lab data set. The results shown in this paper are aligned with the results where we also observed that the image registration method based on the linear curvature gives a better result than the joint model [6, 9].

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Pawar et al. then introduced an atlas-based JSR model based on level set formulation using truncated hierarchical B-splines (THB-splines) [11]. They incorporate atlas-based technique into the segmentation task to overcome noisy and low-resolution images. In this model, B-splines were applied to represent both the level set function as a segmentation result and the spatial transformation mapping. It is then tested in 2D brain MRI and 3D lung MRI. The authors assert that the deformation field represented by THB-splines is said to produce more realistic and smoother deformations than in [9]. In the same year, Debroux et al. initiated a JSR model that combines the hype elasticity framework with Potts model for segmentation process, allowing partition of multiple regions [12].

In 2020, Ademaj et al. proposed a JSR model where an optimization-based approach was introduced through linear curvature combination with weighted Heaviside mutual information (MIH) into the joint segmentation and registration model for multi-modal images deformation [13]. From this paper, the joint model that applies MIH has larger deformation and gives more accurate segmentation and registration results compared to linear curvature based on the MI and NGF models in [14]. Besides, Debroux et al. upgraded their joint model to a space-time-dependent [12, 15]. The model employed a nonlinear elastic regularize with a fidelity measure of shape matching on hepatic DCE-MRI images by utilizing nonlocal share descriptors from the piecewise constant Mumford-Shah model. To optimize the problem, the authors incorporated the time variable and Sobolev space into the model. Ibrahim et al. extended the work in [13] using a modified normalized gradient field functional to produce the joint model for multi-modal images [16].

In this paper, the 2D images data are used for the ease of computation and comparison. The contribution from this paper is.

(i) Comparison of the separate tasks of segmentation and registration with JSR-LC model from Ibrahim et al. for synthetic images with no noise and two levels of the Gaussian noise (low and high) [6].

(ii) Comparison of the performance of the two models: JSR-LC and JSR-B by Pawar et al. for mono-modal real images [11]. We used the 2D mono-modal Mouse Brain Gene Expression Data. Three performance measures are used: mean squared difference (msd), dice coefficient (dc) and the computing time (s).

The outline of this paper is as follows. In the next section, we review the existing Chan and Vese segmentation model, the linear curvature image registration model and two JSR models: JSR-LC and JSR-B. Then, comparisons were made using three experiments and performance metrics. In the last section, the conclusion and future works are provided.

2.0 MATHEMATICAL PRELIMINARIES

Let T, R denote the template and reference images respectively. We are looking for the transformation $\varphi(x)$ aiming for $T(\varphi(x)) = R(x)$. $T(\varphi(x))$ is denoted as the transformed template image. $\varphi(x) = x + u(x)$ where $u(x)$ is the displacement field. The images are assumed to consist of two regions, so-called a two-phase segmentation model. The two-phase segmentation model consists of background and foreground. The level set formulation as in [10] are used for the segmentation of the images.

2.1 Two Phase Chan-Vese Segmentation Model (S-CV)

One of the famous region-based models for variational image segmentation is the Chan Vese model [10]. The model is powerful and flexible that can segment many types of images and widely used in the field of medical image processing [17]. The given image, I is defined on a two-dimensional domain $\Omega \in \mathbb{R}^n$ where $n = 2$ for 2D images. The target contour Γ is represented by the zero-level set of a Lipschitz function $\phi: \mathbb{R}^2 \to \mathbb{R}$, such that.

$$\Gamma = \partial\Omega = \{(x, y) \in \Omega | \phi(x, y) = 0\},$$

$$\text{inside}(\Gamma) = \Omega_1 = \{(x, y) \in \Omega | \phi(x, y) > 0\},$$

$$\text{outside}(\Gamma) = \Omega_1 = \{(x, y) \in \Omega | \phi(x, y) < 0\}$$

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A regularized version of the Heaviside step function is used so that the functional is differentiable and smooth where

$$H_\varepsilon(\phi) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{\phi}{\varepsilon}\right)\right), \delta_\varepsilon(\phi) = H'_\varepsilon(\phi) = \frac{\varepsilon}{\pi(\varepsilon^2 + \phi^2)}$$

The functional for the model is

$$\text{Min } S^{CV}(\phi, c_1, c_2) = \mu \int_\Omega \delta_\varepsilon(\phi)\sqrt{|\nabla H_\varepsilon(\phi)|^2 + \beta}\,dxdy + \int_\Omega (I - c_1)^2 H_\varepsilon(\phi)dxdy + \int_\Omega (I - c_2)^2(1 - H_\varepsilon(\phi))dxdy \quad (1)$$

Once the level set function ϕ is found, the segmented image is given by

$$\text{Seg}(I) = c_1 H_\varepsilon(\phi) + c_1(1 - H_\varepsilon(\phi))$$

Let c_1 and c_2 fixed, the minimizer of (1) with respect to ϕ is found using the the Gateaux derivatives to obtain the following Euler-Lagrange

$$\mu\delta_\epsilon(\phi)\nabla\cdot\left(\frac{\nabla\phi}{\sqrt{|\nabla\phi|^2 + \beta}}\right) - \delta_\epsilon(\phi)(I - c_1)^2 + \delta_\epsilon(\phi)(I - c_2)^2 = 0, \text{in } \Omega \quad (2)$$

with Neumann boundary condition $\frac{\partial\phi}{\partial n} = 0$ on $\partial\Omega$ as in [18]. The minimizer of (1) with respect to c_1 and c_2 where ϕ is fixed is given by

$$c_1 = \frac{\int_\Omega IH_\varepsilon(\phi)dxdy}{\int_\Omega H_\varepsilon(\phi)dxdy}, c_2 = \frac{\int_\Omega I(1 - H_\varepsilon(\phi))dxdy}{\int_\Omega 1 - H_\varepsilon(\phi)dxdy} \quad (3)$$

The PDE in (2) is solved by introducing an artificial time step t and using the gradient descent method to obtain the following evolution equation:

$$\frac{\partial\phi}{\partial t} = \mu\delta_\epsilon(\phi)\nabla\cdot\left(\frac{\nabla\phi}{\sqrt{|\nabla\phi|^2 + \beta}}\right) - \delta_\epsilon(\phi)(I - c_1)^2 + \delta_\epsilon(\phi)(I - c_2)^2 \text{ in } \Omega, \phi(t, x, y) = \phi_0(x, y), \frac{\partial\phi}{\partial n} = 0 \text{ on } \partial\Omega \quad (4)$$

The finite difference scheme and discretization implement of the Equation (4) can be found in [18]. The algorithm for S-CV can be summarized as follows:

Algorithm 1. Chan-Vese algorithm for two phase segmentation

Initialization: Given $\phi_{i,j}^0 = \phi_{i,j,0}$,
For $k = 1{:}\,maxit$ do
Compute c_1 and c_2 using equation (3)
Solve the PDE in equation (4) to update $\phi_{i,j}^k$
If $\left\|\phi_{i,j}^k - \phi_{i,j}^{k-1}\right\| < tol$, set $\phi_{i,j}^k \leftarrow \phi_{i,j}^{k-1}$, break;
End for

2.2 Image Registration Model Using Linear Curvature (R-LC)

The functional image registration model using linear curvature is a second order model that is superior compared to the first order model such as the diffusion and linear elastic models [14]. This is because the linear curvature model can solve affine problems such as global rotation and translation. Thus, there is no need for the affine pre-registration for the linear curvature model. In addition, the model requires

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smoothness in terms of second derivatives. Thus, the model leads to smoother deformation compared to the first order models. The model is given by

$$\text{Min } R^{LC}\big(u(x)\big) = \int_{\Omega} \Big(T\big(x + u(x)\big) - R(x)\Big)^2 \,dxdy + \alpha \int_{\Omega} (\Delta u_1(x))^2 + (\Delta u_2(x))^2 dxdy \tag{5}$$

The resulting Euler-Lagrange equation for the R-LC model is given by

$$\alpha\Delta^2 u(x) + (T(x + u(x) - R(x))\nabla T(x + u(x)) = 0 \tag{6}$$

with boundary conditions $\nabla u_l \cdot n = 0, l = 1,2$. We opted for the discretize then optimize approach to solve the functional in Equation (5). However, the Equation (6) may also be used to obtain the deformation field using the additive operator splitting scheme (AOS). The multilevel approach is used for an efficient and faster implementation of the model. The algorithm for the R-LC model is as follows:

Algorithm 2. Image Registration model using Linear Curvature

Initialization: $R, T, \alpha, minlevel, maxlevel, u^{0,minlevel} = 0$
For level = minlevel,…,maxlevel
(i) Calculate the displacement field u using the quasi-Newton method and Equation (5)
(ii) If level<maxlevel, interpolate u to the next finer level.
End for

2.3 Joint Image Segmentation And Registration Model Using Linear Curvature (JSR-LC)

In 2016, Ibrahim et al. proposed a joint image segmentation and registration model using the linear curvature regularization term [6]. The authors used the linear curvature term for the registration functional. The joint functional for the JSR-LC model is given by

$$\begin{aligned} \text{Min } J^{SR-LC}(u(x), c_1, c_2) = &\int_{\Omega} (R(x) - c_1)^2 H_\epsilon(\phi(x + u(x)))dxdy \\ &+ \int_{\Omega} (R(x) - c_2)^2 (1 - H(\phi(x + u(x))))dxdy \\ &+ \int_{\Omega} (T(x + u(x)) - R(x))^2 H_\epsilon(\phi(x + u(x)))dxdy \\ &+\alpha \int_{\Omega} (\Delta u_1(x))^2 + (\Delta u_2(x))^2 dxdy \end{aligned} \tag{7}$$

The algorithm for the model is as follows:

Algorithm 3. Joint Image Segmentation and Registration Model using JSR-LC

Initialization: $R, T, \alpha, minlevel, maxlevel, u^{0,minlevel} = 0$
(1) Segmentation of the template image using Chan-Vese Segmentation model to produce ϕ_0
(2) Coarsen T, R, ϕ_0 to produce $T^{level}, R^{level}, \phi_0^{level}$ using standard coursing method
(3) For $level = minlevel, \dots, maxlevel$
(i) Calculate the displacement field u(x) using equation (7) where c_1 and c_2 are fixed.
(ii) Update c_1 and c_2 using equation (3) where u(x) is fixed
(4) End for
(5) The segmentation of the reference image is given by $\phi(x + u(x))$
(6) The transformed template image is given by $T\big(x + u(x)\big)$

2.4 Joint Image Segmentation And Registration Model Using B-Splines (JSR-B)

In 2019, Pawar et al. proposed a joint image segmentation and registration model based on truncated hierarchical B-splines (THT-Bsplines) [11]. The B-splines level set function $s(\boldsymbol{x})$ is used to represent the template and reference images. $s(\boldsymbol{x})$ is evaluated using the convolution operator between the discrete cubic B-splines and the level set coefficients. The energy functional for the JSR-B model is given by

$$\mathcal{J}\big(f(x), b(x)\big) = \theta_1 \int_{\Omega} \Big[(R(x) - c_1{}^2 H_{\epsilon}\Big(\phi\big(f(x)\big)\Big) + (R(x) - c_2)^2(1 - H_{\epsilon}\Big(\phi\big(f(x)\big)\Big)\Big] d\Omega$$

$$+\theta_2 \int_{\Omega} \big[g^f(x)(T\big(f(x)\big) - R(x))^2 + g^b(x)(R\big(b(x)\big) - T(x))^2\big] d\Omega$$

$$+\lambda_1 \int_{\Omega} \left\|x + V_{f,u}(x)\right\|_2^2 + \left\|x + V_{f,v}(x)\right\|_2^2 + \left\|x + V_{f,w}(x)\right\|_2^2 d\Omega$$

$$+\lambda_2 \int_{\Omega} \left\|x + V_{f,u}(x)\right\|_2^2 + \left\|x + V_{f,v}(x)\right\|_2^2 + \left\|x + V_{f,w}(x)\right\|_2^2 d\Omega$$

$$+\lambda_3 \int_{\Omega} (\big(x + V_f(x)\big)\, o(x + V_b(x) - x)^2 + ((x + V_b(x))o\big(x + V_f(x) - x\big)^2) d\Omega$$

θ_1 and θ_2 are parameters to balance segmentation and registration in the joint framework respectively. The second functional is related to the minimization of the sum of the squared difference (SSD) between the template and reference images. The functions $g^f(\boldsymbol{x})$ and $g^b(\boldsymbol{x})$ are related to the gradient of the images. They control slow down registration near the edges of the images. The third, fourth and fifth functional are the regularization terms to control the smoothness and symmetrical property of the deformation field. The functionality is minimized using the L_2 gradient flow method with respect to the control points associated with the trivariate B-splines basis functions. See [11] for more details on the algorithm and numerical implementation.

3.0 NUMERICAL EXPERIMENTS

In this section, three experiments are performed involving synthetic and real images. Since the images used are mono modal that consist of the same range of intensity values, hence the mean squared difference (msd) is used to quantify the registration results. The msd between the reference and template images is defined as

$$\mathrm{msd} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{N}} \big(\mathrm{T(i)} - \mathrm{R(i)}\big)^2}{\mathrm{N}} \tag{8}$$

where N is the total number of pixels. In all experiment, $\mathrm{N} = 128 \times 128$. A small value of msd indicates a good registration result which corresponds to a better match between the images. Meanwhile, to evaluate the segmentation result, the dice similarity (dc) metric is used. The dc metric between the reference and template images is given by

$$\mathrm{dc} = \frac{2|\mathrm{Seg(T(x))} \cap \mathrm{Seg(R(x))}|}{|\mathrm{Seg(T(x)}| + |\mathrm{Seg(R(x))}|} \tag{9}$$

The value of dc is between 0 and 1, where 1 show that the segmentation between the two images are closed and 0 shows no overlap between the segmentation of the template and reference images. All Experiment were run using MATLAB R2019b on Windows 10 with 16GB RAM and functions in the MATLAB Image Processing Toolbox were used.

3.1 Experiment 1: Image Segmentation And Registration For Synthetic Images

In the first experiment, synthetic images are used, consisting of a triangle and circle inspired by [11]. The images are tested using the Chan-Vese segmentation model (S-CV), the linear curvature image registration model (R-LC) and the joint segmentation and registration model based on the linear curvature (JSR-LC). The goal is to illustrate that all three models work for simple images without noise by successfully solving the segmentation and registration problems.

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(a) (b)

Figure 1. The template (a) and reference image (b) for Experiment 1. The images consist of red curves which show the segmentation results using S-CV

(a) (b)

Figure 2: The difference between the transformed template and reference image (a) and the transformation (b) for image registration using R-LC for Experiment 1

(a) (b)

Figure 3. The difference between the transformed template and reference image (a) and the transformation (b) for image registration using JSR-LC for Experiment 1

(a) (b)

Figure 4. The difference between the segmentation of the reference image by using the S-CV and JSR-LC models in (a) and (b) is the transformed template image using JSR-LC for Experiment 1

For Figure 1, the value of the performance metrics is dc = 0.9384 and msd = 0.1091. The msd value indicates the difference between template and reference images before we started the registration method. Figure 2, the value of msd = 0.0005 and the computing time is 91.3 seconds. One can observe that the value of msd in Figure 1 is decreasing after registration. Since this is only registration task, there is no value for dc. Figure 3 show the value of the performance metrics are dc = 0.9984 and msd = 0.0004. The computing time for the model is 71.1 seconds. The msd value given by the JSR-LC model is lower than R-LC which indicates JSR-LC produced better results than the R-LC model. From the dc value, we can observe that the segmentation of the refence image is close to the segmentation of the template image since the previous value of dc in JSR-LC is 0.9384. As in Figure 4, the segmentation of the reference image

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in the JSR-LC model is obtained using the segmentation of the transformed template image. The value of the msd = 0.0001 for (a) indicates that the segmentation of the reference image provided by the JSR-LC is equivalent to the segmentation result by the S-CV model.

The results for Experiment 1 are shown in Figure 1, Figure 2 and Figure 3 using S-CV, R-LC and JSR-LC respectively. From the images, one can observe that all three models can solve the problem. The smallest value of msd is given by the JSR-LC model (msd = 0.0004) compared with the R-LC (msd = 0.005). The difference is 0.0001 which is considerably small. However, the bottom corner of the triangle produced by JSR-LC was deformed better than the one produced by R-LC. This is shown by a smaller number of white pixels depicted in Figure 3 (a) compared to Figure 2 (a). In addition, the computing time for JSR-LC is lower than the R-LC model. Even though we combine the tasks of segmentation and registration, the computing time is still low. So, the JSR-LC model does not require more computing time as predicted. The JSR-LC model gives a value of $dc = 0.9984$ which is close to 1 (the previous value was 0.9384) and showed that the model can segment the images.

From Figure 4, one obtains the segmentation of the reference image using the deformed level set which comes from the segmentation of the template image and the transformation found using the JSR-LC model. The segmentation of the reference image is assumed to be equivalent to the segmentation of the transformed template image. The msd between the segmentation of the reference image in Figure 1 (a) using S-CV and the one found from JSR-LC is 0.0001 which indicates a very small difference between these two segmentation results.

3.2 Experiment 2: Image Segmentation And Registration For Synthetic Images With Gaussian Noise

For the second experiment, the images in Experiment 1 are used where the Gaussian noise is added to the triangle image. Now, the goal is to deform the triangle into a circle. The MATLAB denoise function is used as follows.

```
var_gauss = (nl*std(T(:)))^2;
T_E2 = imnoise(T,'gaussian',0,var_gauss);
```

where T is the triangle image in Experiment 1. Two values are used for `nl`; `nl` = 1 for the low level of noise and `nl` = 4 for the high level of noise in the triangle image. In this Experiment, highlighted is given to the cases where the separate task of segmentation and registration are disadvantageous compared to the JSR-LC models.

(a) (b)

Figure 5. The reference (a) and template image (b) with the segmentation results shown by a red curve using S-CV for Experiment 2

For Figure 5, the value of the performance metrics is $dc = 0.8055$ and $msd = 0.1417$The low Gaussian noise is added to the images in Experiment 1 to simulate the S-CV to segment the images. For a low level of Gaussian noise, one can perceive that S-CV manages to segment the images. However, for a high level of noise, the S-CV failed to segment the images as shown in Figure 8. In Experiment 2, we changed the reference and template images from Experiment 1. The results of S-CV are shown in Figure 5. The S-CV managed to segment the triangle image correctly when the noise level is low as shown in Figure 5 (b). However, when the noise level is high, the model failed to segment the image as shown in Figure 8 (a). From Figure 8 (a), the S-CV model segmented the noise at the bottom left of the image. This agrees with the observation made in [19] where the authors mentioned that the S-CV model is at a disadvantage when the images contained high noise.

(a) (b)

Figure 6. The difference between the transformed template and reference image (a) and the transformation (b) for image registration using R-LC for Experiment 2 at a low level of noise

Figure 6, the value of msd = 0.0460. The computing time is 121.2 seconds. The R-LC failed to register the images since the triangle did not become a circle. The results for Experiment 2 using R-LC are shown in Figure 6 (a) and (b) for a low level of noise. We can observe that the triangle did not deform into a circle. For the high level of noise, the model also failed to register the images. The accuracy of any registration models can be severely affected by noise [20]. The value of msd before registration is 0.1417 and after registration is 0.0460 which is still high. One of the ways to overcome image registration in the presence of noise is to filter the images before we proceed with any registration models.

(a) (b)

Figure 7. The transformation (a) and the transformed template image (b) using JSR-LC for Experiment 2 with a low noise level

For Figure 7, the values of the performance metrics are dc = 0.8582 and msd = 0.0266. The computing time for the model is 70 seconds. One can observe that the triangle has been transformed into a circle. The results for Experiment 2 using JSR-LC are depicted in Figure 7 (a) and (b). We can observe that the model manages to register the images with a presence of low level of noise. The value of msd is 0.0266 which is lower than R-LC in Figure 6(a). Meanwhile, the computing time for the model is 70 seconds. Similar observation can be made from Experiment 2, that the JSR-LC model does not increase the computing time in solving the problem. The dc value for this model is 0.8583 which is higher than 0.8055 in Figure 5.

(a) **(b)**

Figure 8. A higher level of noise is added to Experiment 2. The failed segmentation of template images using S-CV is shown in (a). However, the joint model JSR-LC still manages to perform image registration where the transformed template image is shown in (b)

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The results for JSR-LC model with the presence of high level of noise are given in Figure 8. The S-CV fails to segment the triangle correctly. However, the JSR-LC manages to register the images.

3.3 Experiment 3: Mouse Brain Gene Expression Data For JSR-LC And JSR-B

In this Experiment, Mouse Brain Gene Expression Data is used to compare the two JSR models: JSR-LC and JSR-B. The data consists of 8 pairs of images with courtesy from the Center for Computational Biology, UCLA. The goal is to map a 2D slice of mouse brain (template image) to its corresponding 2D slice of the mouse brain atlas (reference image). The purpose of this brain atlas mapping is to facilitate the integration of anatomic, genetic, and physiological observations from multiple subjects in a common space [21]. As an example, the reference and template images for pair no. 5 are shown in Figure 9. In this Experiment, $\alpha = 10$ is used for the JSR-LC model. Meanwhile, for the JSR-B, the values of the parameters are $\theta_1 = 100, \theta_2 = 1, \lambda_1 = \lambda_2 = \lambda_3 = 0.01$.

(a) (b)

Figure 9. The reference (a) and template image (b) for the Mouse Brain Gene Expression Data. This is pair no. 5 shown as an example of the images

The results for Experiment 3 are shown in Table 1. One can observe that the JSR-LC model outperformed the JSR-B model for the three performance measures. The average value given by JSR-LC model is lower than the JSR-B model. In addition, the dc values for all pairs given by JSR-LC model are higher compared to JSR-B model. This showed that the JSR-LC model is better than the JSR-B in segmenting and registering the images. The computing time for the JSR-B model is higher than the JSR-LC model, signifies that JSR-LC model is faster than the JSR-B model. Table 1 shows the value of the performance metric for the JSR-LC and JSR-B models which $*$ indicates that the time needed is more than 2500 seconds. We can observe that the JSR-LC model is better than the JSR-B model in terms of segmentation and registration accuracy. In addition, the model is also faster than the JSR-B model based on computing time.

Table 1. The value of the performance metric for the JSR-LC and JSR-B models

Pair No.	msd		dc		Time (s)	
	JSR-LC	JSR-B	JSR-LC	JSR-B	JSR-LC	JSR-B
1	0.0180	0.0801	0.9861	0.8094	67.0	$*$
2	0.0112	0.0710	0.9900	0.8184	45.7	$*$
3	0.0123	0.0699	0.9883	0.8513	67.1	1125.2
4	0.0146	0.0687	0.9811	0.8549	68.0	2362.3
5	0.0170	0.0480	0.9812	0.9037	61.9	1968.8
6	0.0279	0.0258	0.9624	0.9367	53.3	1066.4
7	0.0254	0.0298	0.9536	0.9155	53.8	1062.3
8	0.0150	0.0206	0.9713	0.9287	53.4	1499.1
Average	0.0177	0.0517	0.9768	0.8773	58.8	1514.0

4.0 CONCLUSION

A joint image segmentation and registration model based on the linear curvature (JSR-LC) for mono-modal images is investigated and compared with the separate tasks of segmentation and registration. We also present the comparison of the model with the state-of the art model which is the joint segmentation and registration based on the THB-splines model (JSR-B). We showed an improvement in the accuracy for both the segmentation and registration of 2D images compared to the individual tasks. Based on the test

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images used, the JSR-LC model is better than the JSR-B model in terms of the values of msd and dc. We observed that the success of the joint model is dependent on how well the segmentation results manage to represent the images. This is aligned with the outcome pointed out in [22]. In the current work, we use the Chan-Vese model to perform the segmentation in the joint framework. We plan to extend this to perform multi-phase image segmentation in the presence of high noise and intensity inhomogeneity. We also plan to study selective segmentation models to generalize and extend our joint framework.

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